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A RANDOM MODEL FOR HALF-AMPLITUDE DECAY TIMES OF RAYLEIGH WAVES EXTENDED ARRAY EVALUATION PROGRAM

Stephen A. Alsup, et al

Texas Instruments, Incorporated

Prepared for:

Advanced Research Projects Agency
Air Force Technical Applications Center

25 January 1973

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AFTAC Project No. VELA T/2705/B/ASD

A RANDOM MODEL FOR HALF-AMPLITUDE DECAY TIMES OF RAYLEIGH WAVES

SPECIAL REPORT NO. 4
EXTENDED ARRAY EVALUATION PROGRAM

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TEXAS INSTRUMENTS INCORPORATED
Services Group
Post Office Box 5621
Dallas, Texas 75222

Contract No. F33657-72-C-0725
Amount of Contract: $417,000
Beginning 1 April 1972
Ending 31 March 1973

Prepared for

AIR FORCE TECHNICAL APPLICATIONS CENTER
Alexandria, Virginia 22314

Sponsored by

ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Monitoring Research Office
ARPA Order No. 1714
ARPA Program Code No. 2F10

25 January 1973

Acknowledgement: This research was supported by the Advanced Research Projects Agency, Nuclear Monitoring Research Office, under Project VELA-UNIFORM, and accomplished under the technical direction of the Air Force Technical Applications Center under Contract No. F33657-72-C-0725

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Texas Instruments Incorporated Services Group P.O. Box 5621, Dallas, Texas 75222	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

A Random Model for Half-Amplitude Decay Times of Rayleigh Waves
Extended Array Evaluation Program Special Report No. 4

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Special

5. AUTHOR(S) (First name, middle initial, last name)

Stephen A. Alsup and David G. Lambert

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
25 January 1973	40	3

8a. CONTRACT OR GRANT NO. Contract No. F33657-72-C-0725 b. PROJECT NO. AFTAC Project No. c. VELA T/2705/B/ASD d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

11. SUPPLEMENTARY NOTES ARPA Order No. 1714	12. SPONSORING MILITARY ACTIVITY Advanced Research Projects Agency Nuclear Monitoring Research Office Arlington, Virginia 22209

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DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Signal Persistence						
Rayleigh Waves						
Seismic Network						

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SECTION I

INTRODUCTION

Estimation of the significance of seismic signal interference in the operations of a network of seismic recording stations is of importance both toward understanding signal detection potentials and for evaluating the necessity of development and use of "anti-interference" analytical tools. Use of the seismograms from the Very Long Period Experiment (VLPE) stations provides the opportunity for study of generalized multiple path signal trends in a network sense. Current seismological trends are toward centeralization of operations to the point where world-wide station distributions are treated simply as instrument arrays, and the kind of approach taken here is both necessary and vital to successful use of data provided by the recording stations.

While most who are familiar with seismograms can call to mind some approximate length of time that long-period Rayleigh waves are visible on our recordings, virtually no numerical data on the subject have been published, and no time-amplitude persistence reports are known to us. The subject is of interest upon occasion, particularly if we wish to have a measure of some specific signal at a time when another signal occupies the seismogram. The length of time that some level of amplitude of the "unwanted" signal persists is also of some interest if we wish to forecast the possibility that the signal interference is at a level where the required measurement will be obscured. We hope to provide a method for estimation of the time-amplitude characteristics of long-period Rayleigh wave signals with this study, using signals recorded by the VLPE stations from Eurasian earthquakes as a data base.

SECTION II

THEORY

Stochastic models which describe emission of radioactive particles, the flow of telephone or automobile traffic, mean time between component failure, and other random occurrences of events in time are widely used to provide estimates of event occurrence probabilities. The "event" selected here for analysis is the occurrence of signal amplitude decay to one-half its maximum, the decay of that half-amplitude to one-half of its value, and so on. The time required for this event to occur is the factor we wish to describe. We therefore need to find a model which will fit the observed data and be able to assign numerical values to the model parameters. Such models usually fall in a general class termed "inter-arrival" or "waiting time" stochastic processes (Parzen, 1962, 1967; Hogg and Craig, 1968).

One basic model for waiting times is that used for events of a Poisson type. The model parameter is the rate of occurrence of independent events within successive time intervals of length t. If N_k is the number of times that k events are observed within the interval t, the probability $P(k)$ that exactly k events will be observed in t is found to be:

$$P(k) = \frac{e^{-\lambda t} (\lambda t)^k}{k!} \qquad (1)$$

where λ is the reciprocal of the mean number of events observed in t (i.e., $\lambda = 1/m$ if m is the mean of observed times) or the event/unit time rate of occurrence ($= \sum N_k(k)/t$). The model may be used to describe the expected time for an event to occur as well as the probability of an event occuring within some discrete time interval in a succession of intervals. The probability $P(k)$ that at least k events will be observed in a time interval is

$$P(k) = P(n) + P(n+1) + P(n+2) + \ldots + P(n+m) \tag{2}$$

$$= \sum_{n=k}^{n+m} \frac{e^{-\lambda t} (\lambda t)^k}{k!} \qquad n = k, k+1, k+2, \ldots$$

and the probability $P[k]$ that no more than k will be observed in the interval is

$$P[k] = 1 - P(k) \tag{3}$$

For this study we know that an event has occurred, i.e., that half-amplitude was reached in our observations simply because we have a measurement (or several measurements). The Poisson process lead to use of the exponential probability law to describe our data, or more generally, gamma probability functions related to the Poisson process. If the observed half-amplitude decay times, which we will now call W_n, follow a Poisson process of occurrence at a mean rate of λ events/unit time, then the occurrence of the r^{th} event ($r = 1, 2, 3, \ldots$) obeys a gamma probability law with parameters r and λ. This law has a probability density function $f(t)$ of:

$$f(t) = \frac{\lambda}{(r-1)!} (\lambda t)^{r-1} e^{-\lambda t} \qquad t \geq 0 \tag{4}$$

$$= 0 \qquad t < 0$$

The waiting time (or decay time) to the first event (from peak amplitude A to A/2, or A/2 to A/4,...) then follows the gamma probability law with parameter $r = 1$ and:

$$f(t) = \lambda e^{-\lambda t} \tag{5}$$

For $t \geq 0$, the probability that the waiting time to half-amplitude will be less than or equal to t (or that the interval t will include at least one half-amplitude) is the integral

$$P\left[W_n \leq t\right] = \int_0^t \lambda e^{-\lambda t} dt \qquad (6)$$

and the probability that it will be greater than t (or that the interval t will hold no half-amplitude) is

$$P\left[W_n \geq t\right] = 1 - P\left[W_n \leq t\right] = \int_t^\infty \lambda e^{-\lambda t} dt \qquad (7)$$

For the problem of describing successive half amplitude times after peak amplitude A, one easily evaluated extended form of (7) is given by Parzen (1967):

$$P\left[W_n \geq t\right] = \sum_{r=0}^{j-1} \frac{1}{r!} (\lambda t)^r e^{-\lambda t} \qquad r=0,1,2,\ldots,j-1 \qquad (8)$$

where the probability distribution of occurrence times is described for the first half-amplitude A/2 at $r = 0$, for the second half-amplitude A/4 at $r = 1$, for the third half-amplitude A/8 at $r = 2$, and so on.

In the following sections the distribution of all half-amplitude times observed - i.e., the time between peak amplitude A and A/2, between A/2 and A/4, etc. - will be tested as a Poisson distributed random variable. We will show that goodness of fit tests for observed and predicted W_n distributions have high probability results. These results indicate that the W_n are independent exponentially distributed random variables of a Poisson type simply because the Poisson process is characterized by this fact (see Parzen, 1962). The demonstration that this type of distribution is satisfied will allow a prediction of the distribution of times between A and successive amplitude increments, varying only the parameter r with λ held constant (Equation 8). A model such as equation 8, if fitted successfully, would allow prediction of the expected time of signal persistence on the recordings as well as the amplitudes expected during the continuation of signal and coda.

SECTION III

DATA AND PROCEDURES

A. SOURCE OF DATA

Data for the study were recorded at VLPE stations listed in Table III-1 (also shown in Figure III-1) during January through March of 1972. Paper record seismograms, analog conversions of digital magnetic tape recordings, and bandpassed digital conversions to analog seismograms were used to obtain measurements of signal amplitude and time data. Typical response characteristics of the recording channels are shown in Figure III-2. Bandpassing of the digital recordings was applied in the frequency domain, retaining data in the 19.4 to 42.5 second period range. Only vertical component data channels were analyzed, and only those Rayleigh wave signals which appeared free of interference from other earthquakes were used since we were attempting to isolate the basic decay patterns of the waveform. Preliminary Determination of Epicenter (PDE) lists were used for hypocenter parameters. The general location of sources studied are also shown in Figure III-1.

B. ANALYSIS PROCEDURES

Long-period Rayleigh wave signals from these sources were analyzed for "half-amplitude decay times" by the following steps:

1. The time T_1 between the maximum recorded Rayleigh wave amplitude A (peak to peak, regardless of wave period) and the latest recorded signal amplitude equal to A/2 (also peak to peak, regardless of wave period) was measured to the nearest 10 seconds and recorded with distance, station, bodywave magnitude, and recording system for each signal analyzed.

TABLE III-1

VERY LONG PERIOD EXPERIMENT (VLPE)
STATIONS AND LOCATIONS

Station	Designator	Latitude	Longitude
Charters Towers, Australia	CTA	20.1S	146.3E
Chiang Mai, Thailand	CHG	18.8N	99.0E
Fairbanks, Alaska	FBK	64.9N	148.0W
Toledo, Spain	TOL	39.9N	4.0W
Eilat, Israel	EIL	29.3N	34.5E
Kongsberg, Norway	KON	59.6N	9.6E
Ogdensburg, New Jersey	OGD	41.1N	74.6W
Kipapa, Hawaii	KIP	21.4N	158.0W

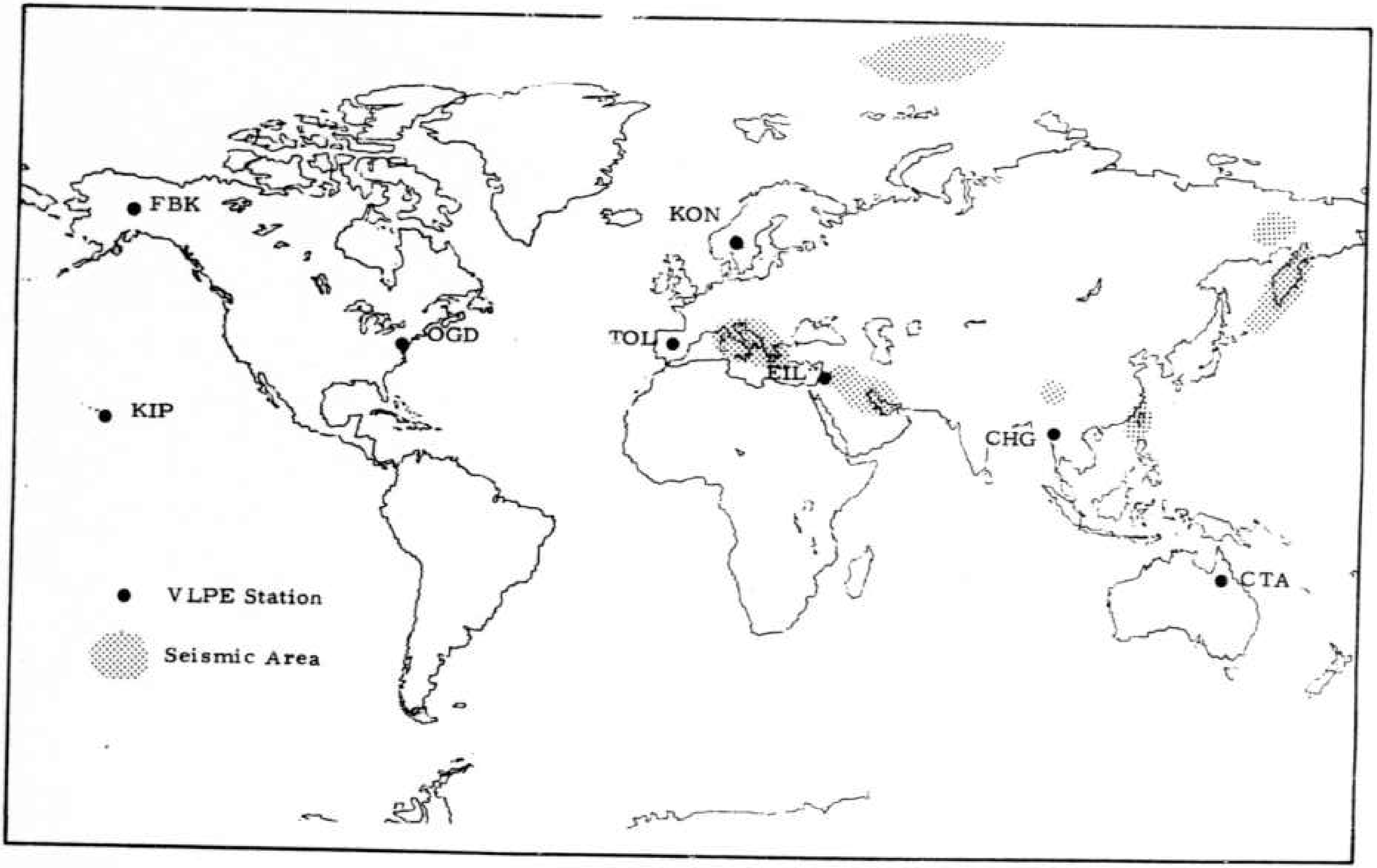


FIGURE III-1. Very Long Period Experiment (VLPE) Station Locations and Seismic Source Areas for Half-Amplitude Rayleigh Wave Decay Signal Analysis.

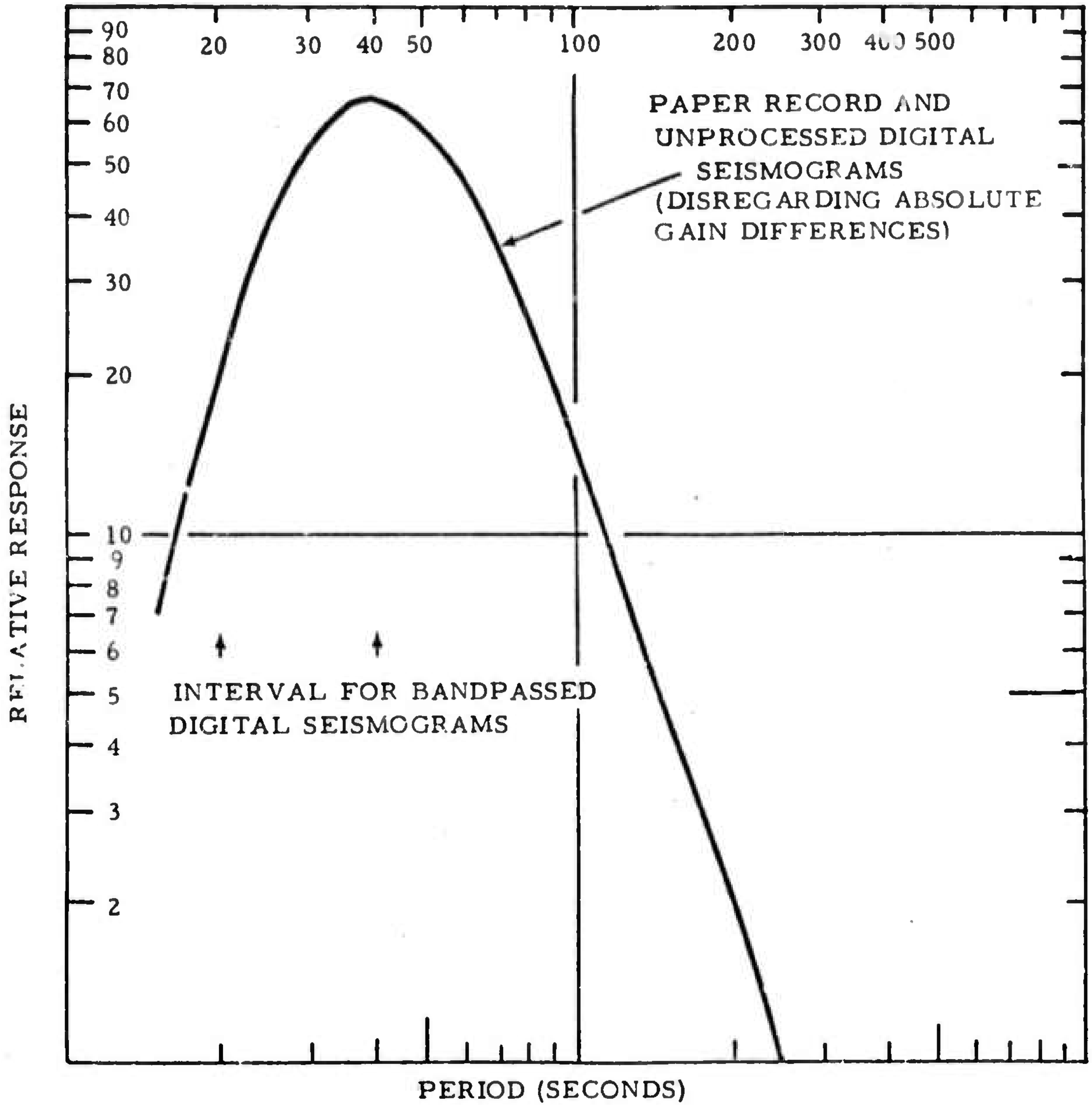


FIGURE III-2

Typical VLPE Station Vertical Channel Response Characteristics

2. Times for successive half-amplitudes (T_2, T_3, etc.) for A/4, A/8, etc., respectively, were also measured to within 10 seconds to the place on the recording where the signal waveform or definite signal-related coda could not be distinguished from the ambient seismic noise. The approximate time of signal loss in the noise was also recorded if the half-amplitude signal step could not be obtained at that point.

Several examples of the measurements are shown in Figures III-3, and III-4 for a signal source in Iran as recorded by stations in Norway, Spain, New Jersey, and Alaska.

C. DISCUSSION

The first time interval T_1, the difference between T_1 and T_2, T_2 and T_3, and so on, represent the "half-amplitude decay times" (or waiting times W_n to half-amplitude) that we intend to model. These times are a measure of Rayleigh wave persistence viewed through the recording channels which may have characteristics suitable for modeling.

The problem of trying to characterize half-amplitude decay is readily seen in Figures III-5, III-6, and III-7 where the observed T_1, T_2, T_3, and so on, are plotted versus their respective amplitude increments. Observation of time to noise, T_n, is plotted half-way between the last observable amplitude increment before signal loss and the next increment in the figures. The plotted data are restricted to observations of less than 2200 seconds total between peak amplitude and observation time to allow some detail of the main bulk of data.

While not obvious in the display of data in Figures III-5, III-6, and III-7, there are no consistent linear trends in the signal decays related to distance. An exponential randomness will be shown later which apparently dominates the amplitude decay trends at all distances. A weak tendency for more rapid A/2 decay at close distance is evident, and most of the A/8 and

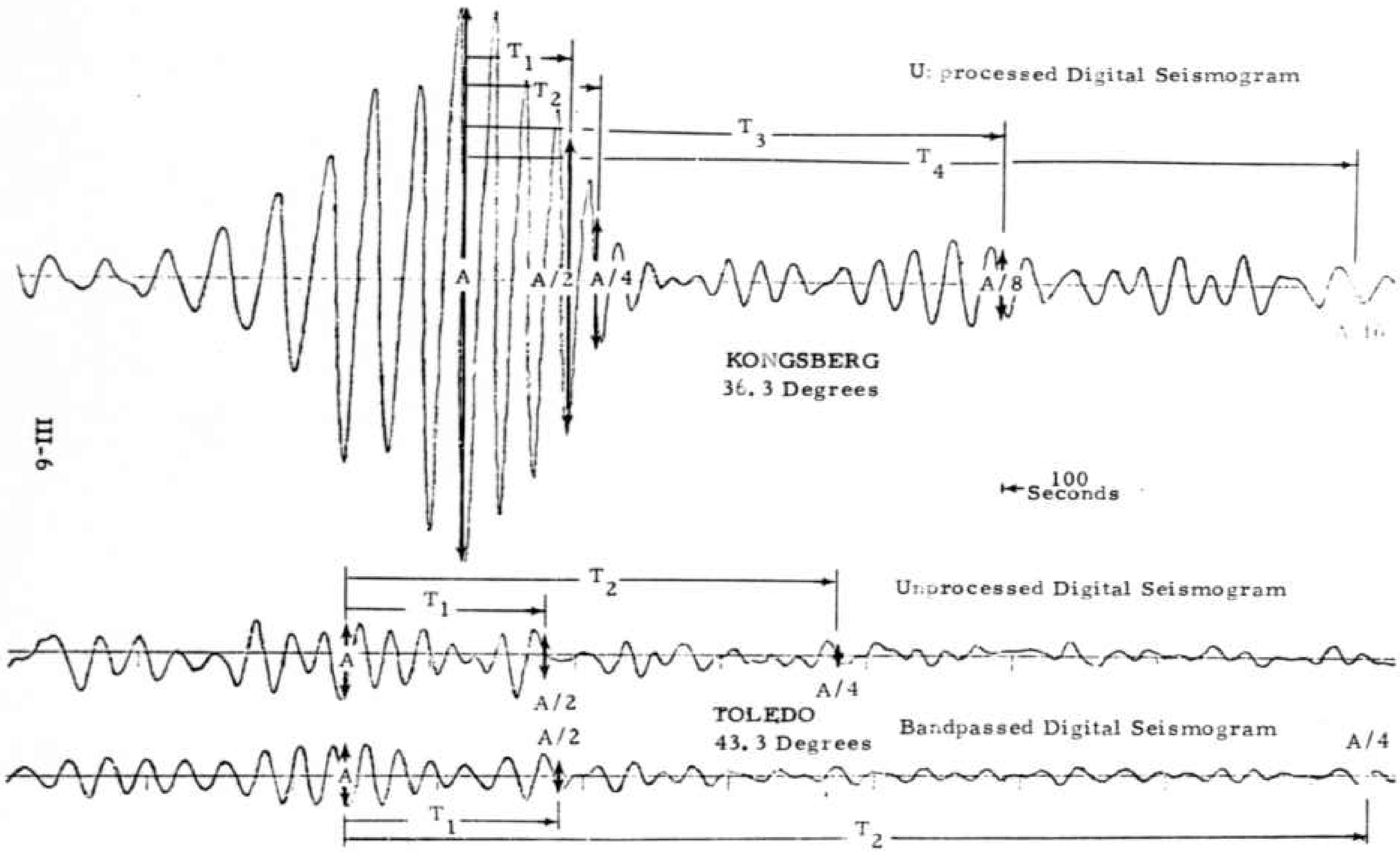


FIGURE III-3. Measurement Procedure for Half-Amplitude Decays of Long Period Rayleigh Wave Signals. Epicenter 35.0N 51.0E on February 3, 1972, $m_b = 5.0$ (C&GS)

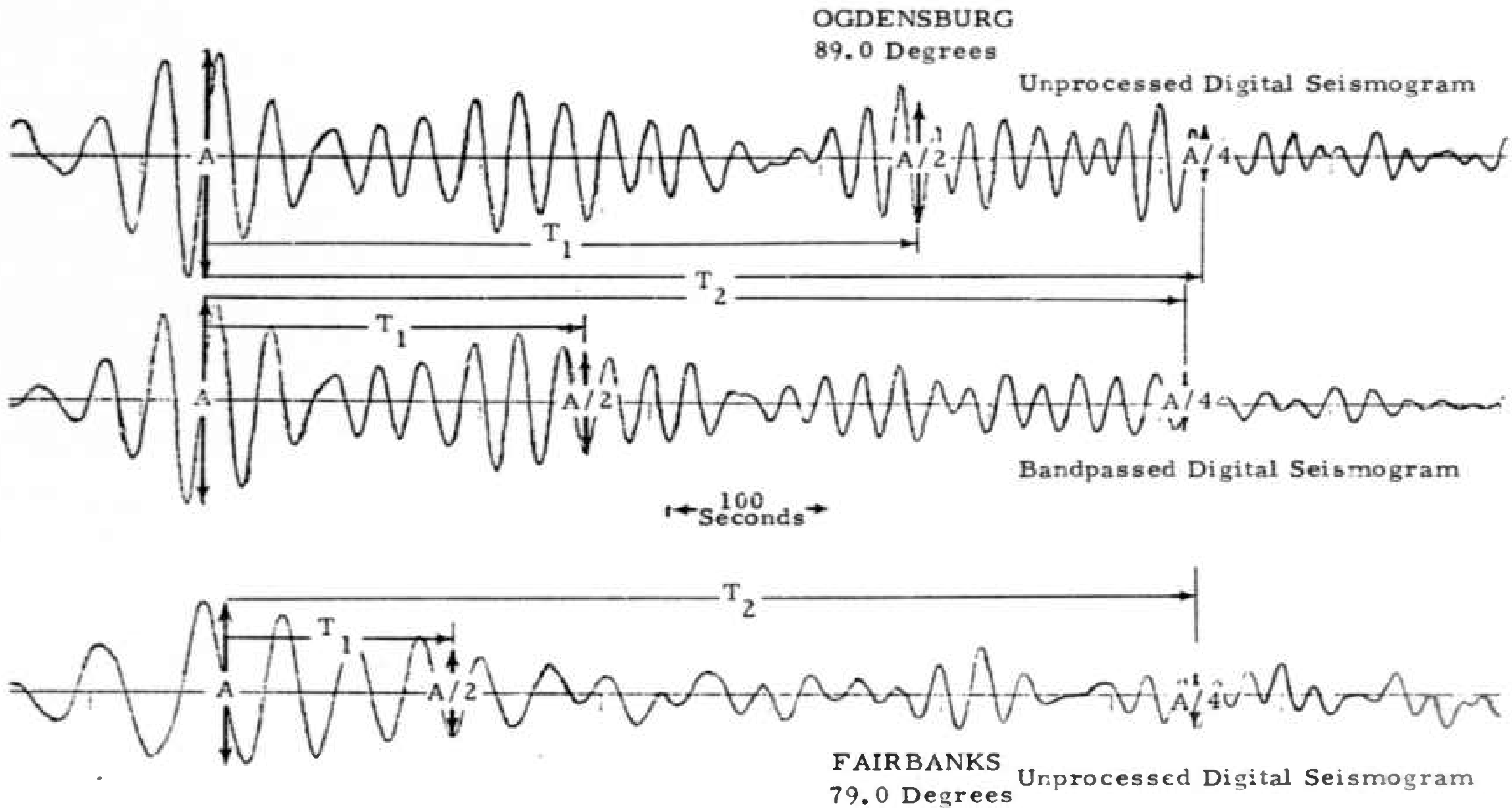


FIGURE III-4. Measurement Procedure for Half-Amplitude Decays of Long Period Rayleigh Wave Signals. Epicenter 35.0N 51.0E on February 3, 1972, m_b = 5.0 (C&GS)

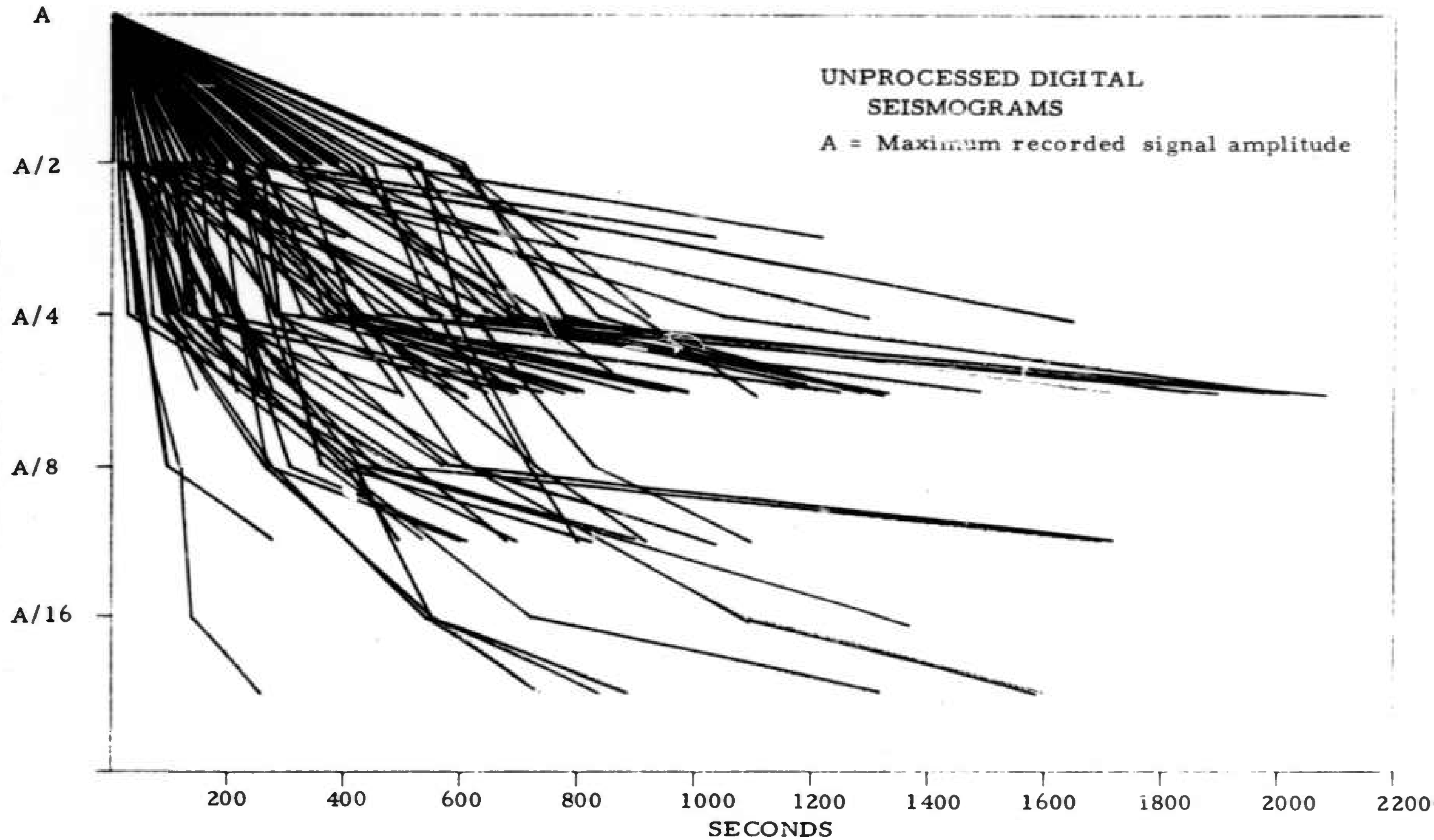


FIGURE III-5. Typical Half-Amplitude Rayleigh Wave Decay Patterns. Times Indicate Occurrence Of Half-Amplitude Increments.

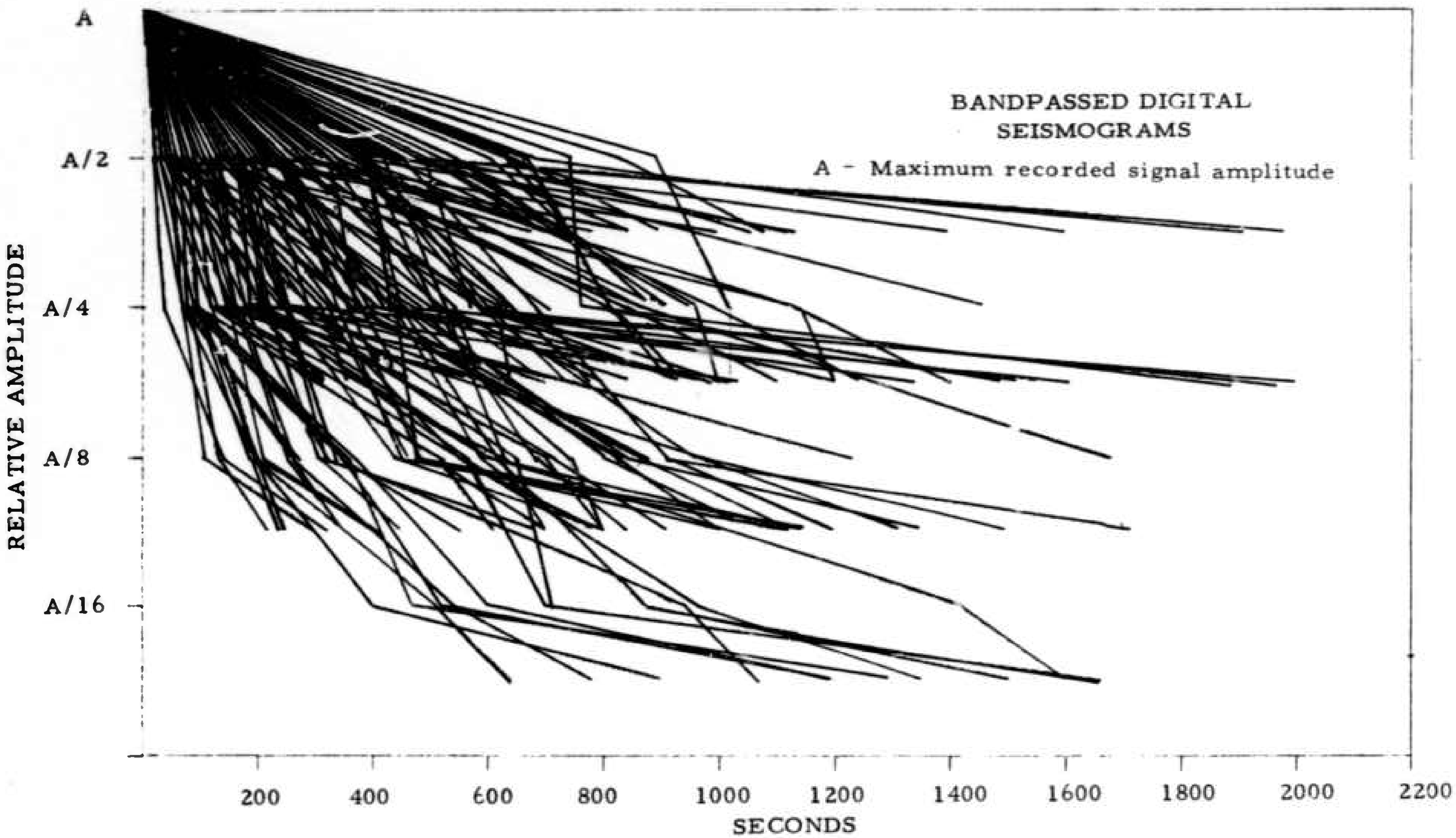


FIGURE III-6. Typical Half-Amplitude Rayleigh Wave Decay Patterns. Times Indicate Occurrence Of Half-Amplitude Increments.

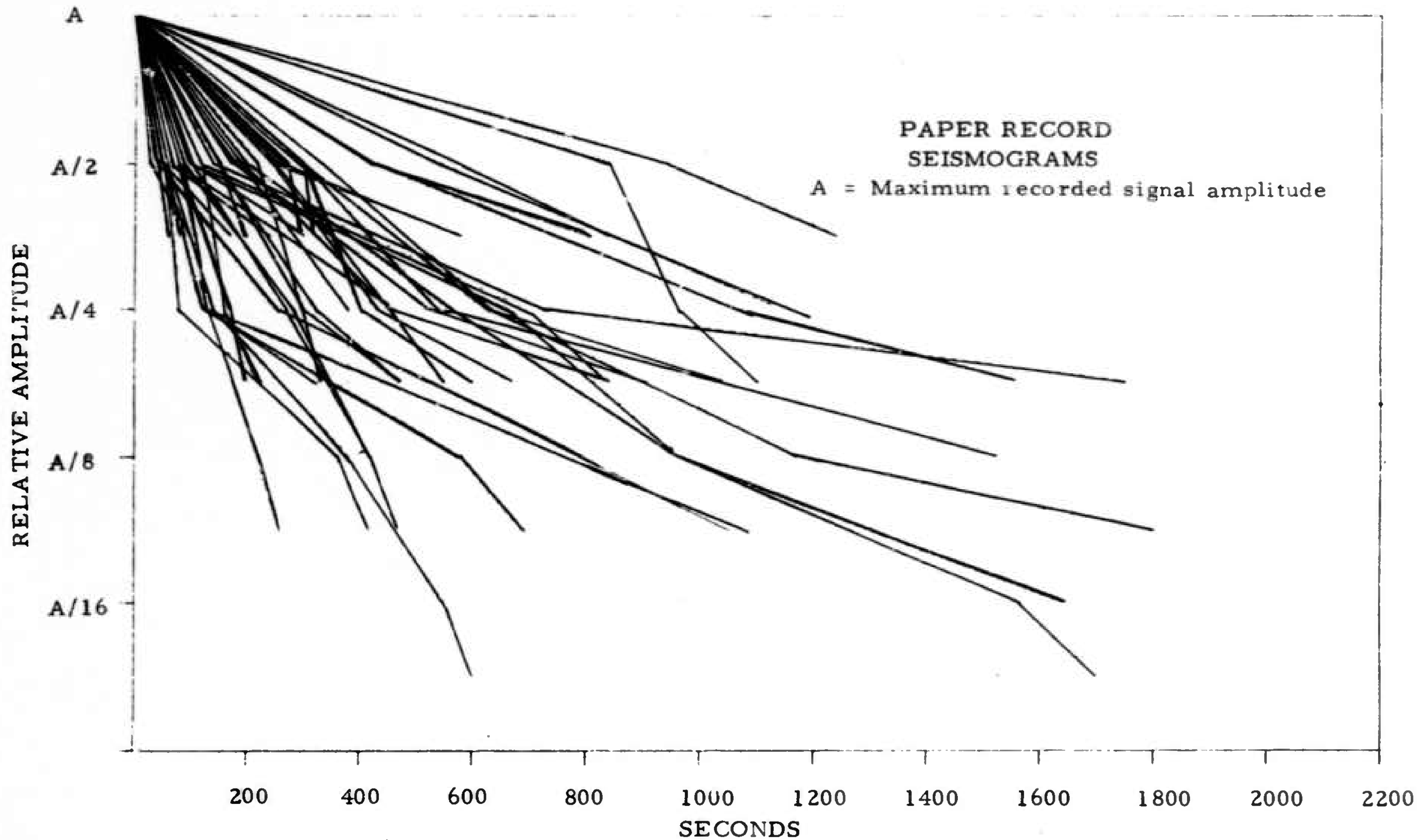


FIGURE III-7. Typical Half-Amplitude Rayleigh Wave Decay Patterns. Time Indicate Occurrence Of Half-Amplitude Increments.

A/16 measurements were taken at epicentral distance less than 50 degrees. Difference in signal dispersion for many paths, the wave period of peak amplitude relative to both source magnitude and period-dependent propagation factors, plus a variety of multipathing contributions to the main signal and coda, all contribute to randomization of the half-amplitude times. Randomness in the observations is present in all parts of the plots, however, with a wide range of decay times observed for the A/2 increment, both rapid and slow decay to A/4, and rapid and slow decay for A/8, etc. Such randomness might be expected since we have included a very wide range of wave paths, distances, and magnitudes as a goal for describing the generalized character of the waveforms observed in a network of recording stations. We could also expect, and we do see, definite non-random amplitude decays for specific source-station paths.

No clear relationship was found for a magnitude-half-amplitude trend for either network or single path observations (magnitudes range from $m_b = 3.2$ to $m_b = 6.2$). In fact, amplitude decay patterns for specific paths tended to be very similar (within 30 to 40 seconds for half-amplitude increments) over at least one order of magnitude, and paths repeated by signals from about the same magnitude source produce very nearly identical decay patterns. Some tendency to have longer first half-amplitude (A/2) decay time with increasing magnitude was noted for specific paths (as was a tendency to have a longer wave period for peak amplitude), but this was not a clear trend in all of the data observed. A lack of very large events ($m_b = 7.0$ or greater) prevented evaluation of the signal characteristics of very long-term, multiple-path surface wave signals which are well known in observational seismology.

Distance distribution of the observations (number of data channel source pairs) is shown in Figure III-8 for the entire data set. An average of about two half-amplitudes were obtained from each pair, and a maximum of four times (A/16) was the practical measurement limit. Time to noise, T_n, provided

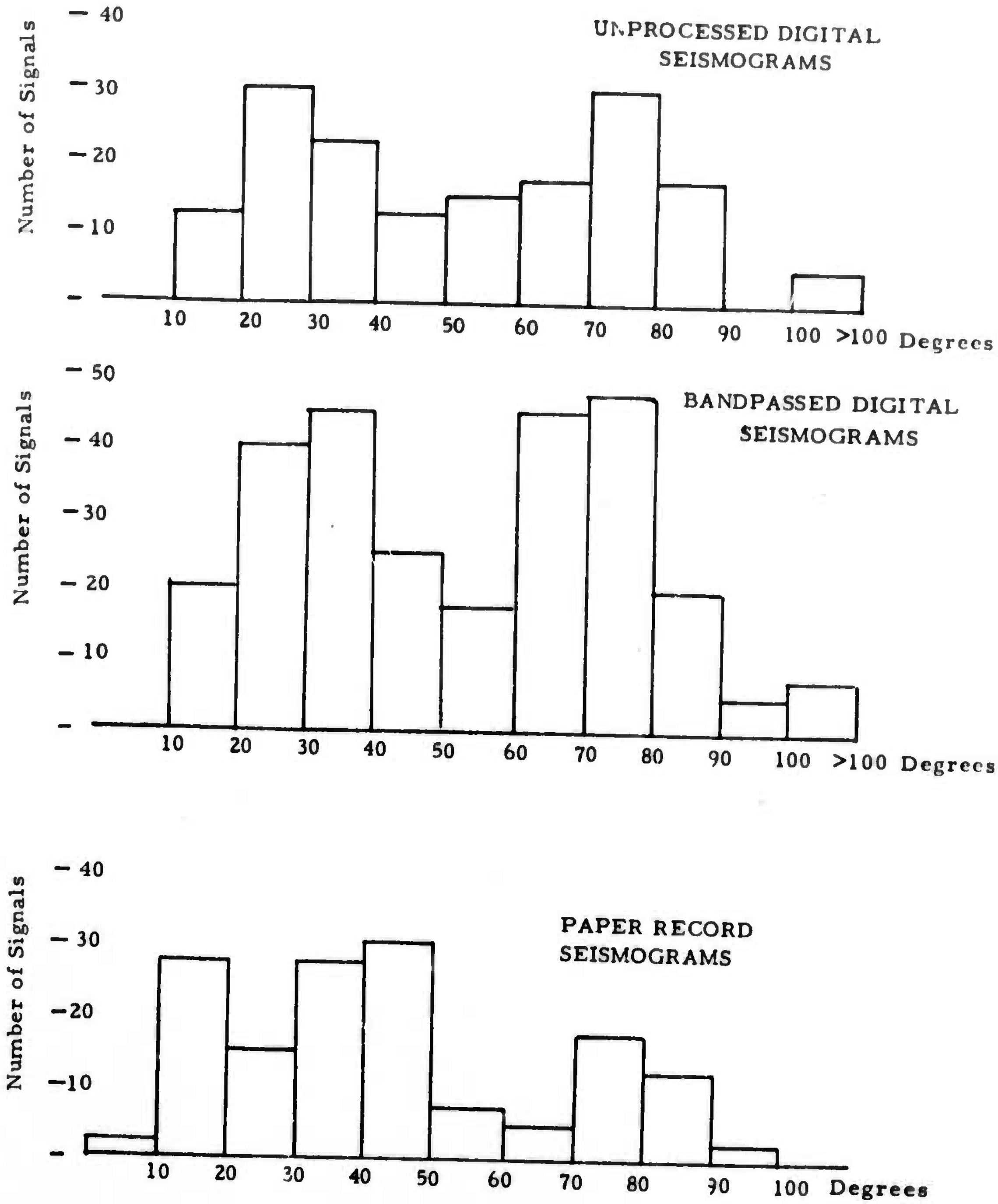


FIGURE III-8

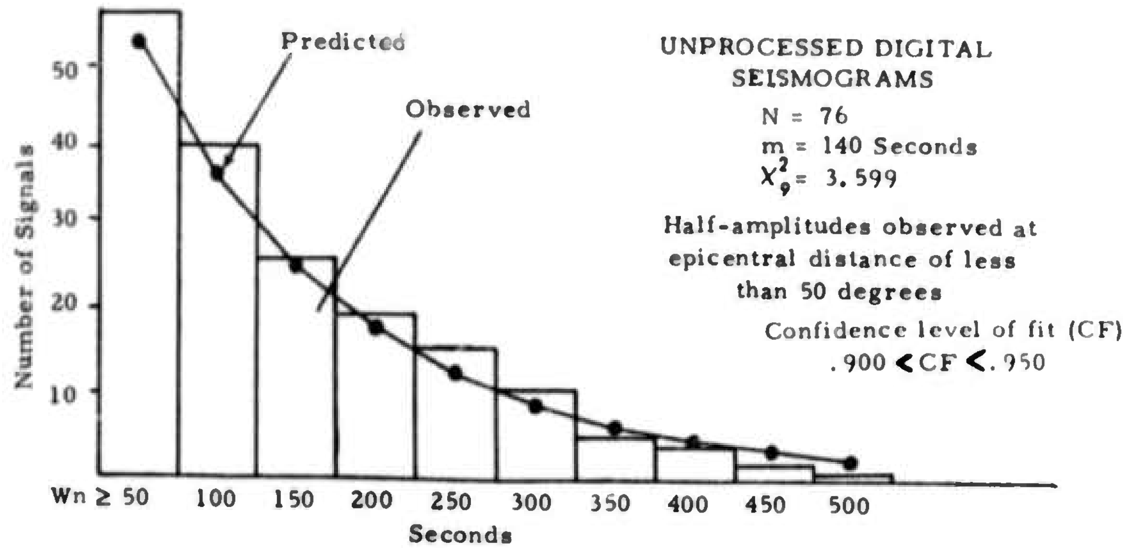
Number of Rayleigh Wave Signals Analyzed Versus Epicentral Distance for Half-Amplitude Decay Model

the possibility of a fifth measurement, but this is not generally included in the modeling unless the amplitude of noise equaled one of the signal increments. Because of the bimodal characteristic of the data sample, a separation for analysis was made by modeling events from either greater or less than 50 degrees of epicentral distance.

SECTION IV
RESULTS

The observed half-amplitude decay times from the VLPE recordings were summarized so that the number of observations with times greater than successive 50 second multiples are plotted and compared with the number of events predicted by the gamma distribution given in equation 7. The observed and predicted number of events are shown in Figures IV-1, IV-2, and IV-3. For the comparison, the gamma parameter $r = 0$, and the parameter λ = the number of observations divided by sum of all observed decay times (or, equivalently, the reciprocal of the mean decay time). The predicted number of events is the product of the total number of observations (N) and $P\left[W_n \geq t\right]$, with $P\left[W_n \geq t\right]$ evaluated at $t = 50, 100, 150, \ldots$ seconds. Differences in the λ term (or 1/m) at each distance range shown in Figures IV-1, IV-2, and IV-3 are most likely due to the different data bases used to obtain the half-amplitude time measurements. Little duplication of signals between the paper records and digital seismograms is represented in the observations. Unprocessed and bandpassed signal analysis was performed essentially on the same signal base, but only 60 - 70% of the times observed on the bandpassed recordings could be measured on the unprocessed recordings. Difficulty in measuring second, third, and fourth half-amplitude increments in the wider band noise of the unprocessed recordings is the primary cause of differences in the latter two cases.

The data are shown for signals observed at less than or more than 50 degrees of epicentral distance because of the bimodal characteristics of the observations noted earlier (Figure III-8). If the data are shown to fit a random model in both instances, they will also fit a model which includes



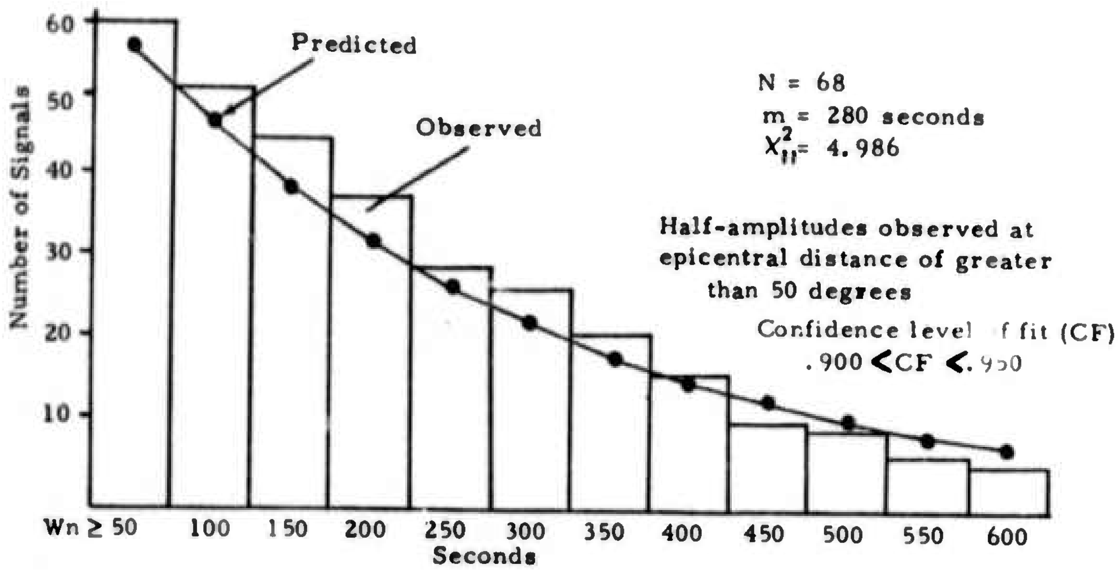


FIGURE IV-1

Predicted and Observed Gamma Distributed Waiting Times (Wn) For All Observed Half-Amplitude Rayleigh Wave Signal Amplitude Decays. Gamma Distribution Parameters r = 0 and Intensity 1/m.

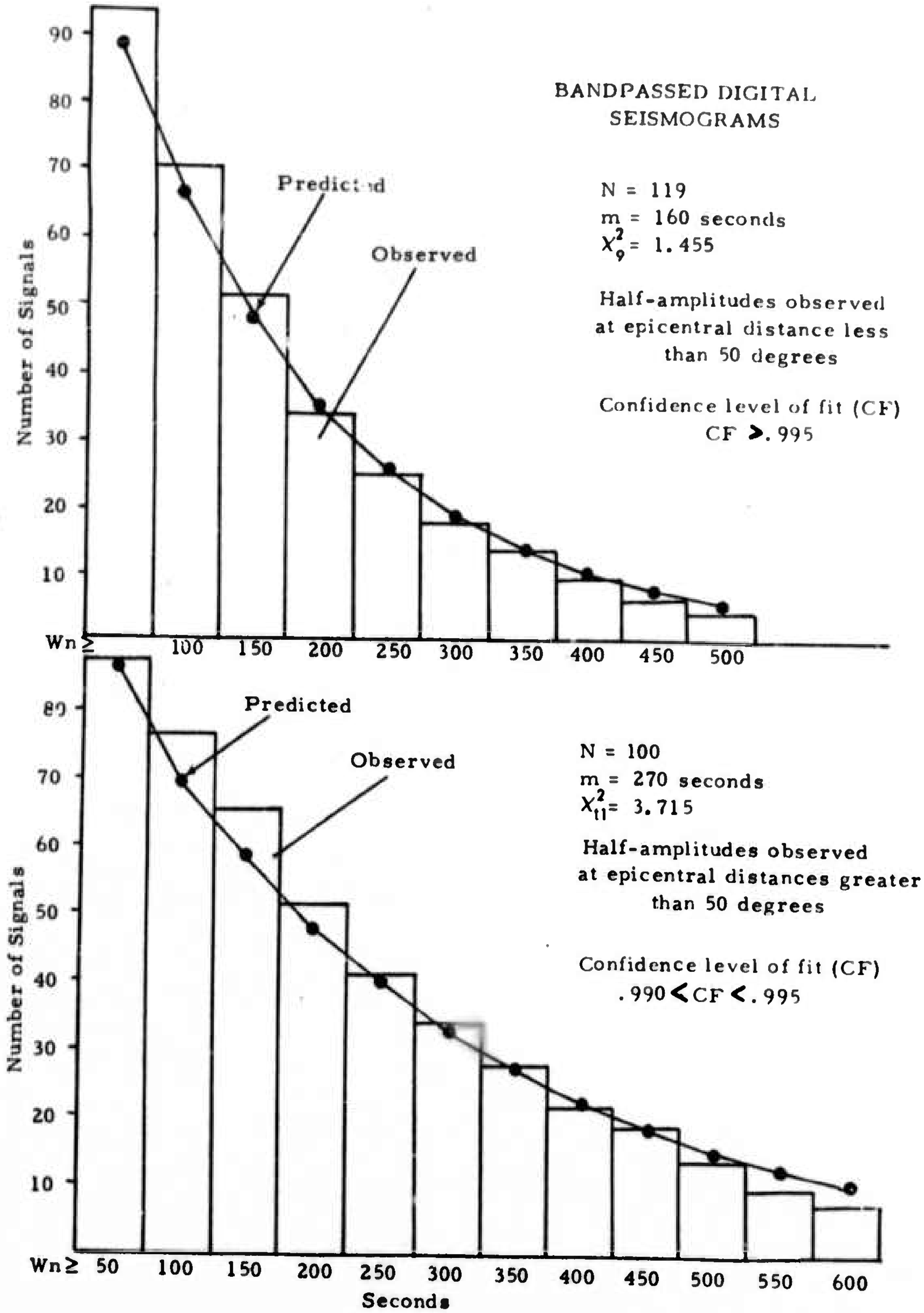


FIGURE IV-2

Predicted and Observed Gamma Distributed Waiting Times (Wn) For All Observed Half-Amplitude Rayleigh Wave Signal Amplitude Decays. Gamma Distribution Parameters are r = 0 and Intensity 1/m.

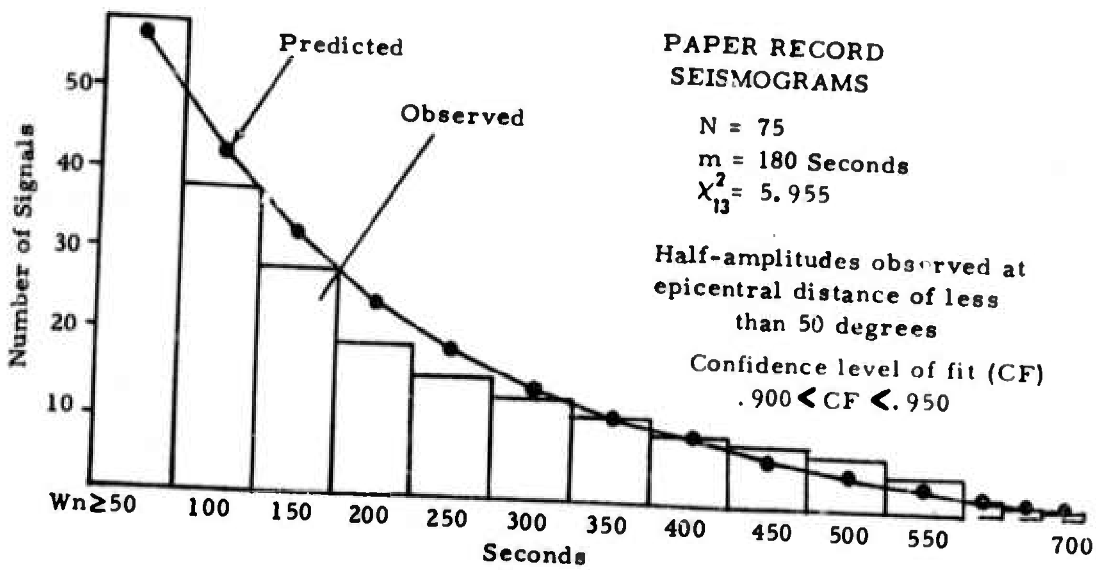


FIGURE IV-3

Predicted and Observed Gamma Distributed Waiting Times (Wn) For All Observed Half-Amplitude Rayleigh Wave Signal Amplitude Decays. Gamma Distribution Parameters $r = 0$ and Intensity $1/m$.

all observations if the parameter λ is changed appropriately. Theorems regarding the randomness of subsets of observations taken from distributions in the exponential class of probability density functions (such as the Poisson) require the foregoing conclusion. In fact, the complete sufficient statistic for the entire data set in each case is $\lambda t = (N_1 \lambda_1 t + N_2 \lambda_2 t)/ (N_1 + N_2)$ if N is the total number of observations and the subscripts 1 and 2 refer to data observed at less than or more than 50 degrees, respectively. Fits to the total data set for the bandpassed signals and unprocessed signals (neither shown) for the A/2 data alone using the statistic above are satisfactory at a confidence level greater than .950 and greater than .900, respectively. Insufficient data to attempt a model fit were found in the paper record analysis for events occurring beyond 50 degrees of epicentral distance, which is a further reason for maintaining the separation of observed data as shown here.

The "goodness of fit" for the models and respective data sets was tested by the Chi-squared procedure in which the predicted and observed number of events (N) were compared by

$$\chi^2_{n-1} = \sum_n \frac{(N \text{ pred.} - N \text{ obs.})^2}{N \text{ pred.}} \tag{9}$$

for n of the 50 second multiple intervals. As is customary for the procedure, data for later intervals which were rarely observed are lumped together in the last (longest time) interval. The values of the calculated χ^2 was then compared to a standard table of χ^2 values for one less degree of freedom than the number of intervals tested. If the calculated value of χ^2 exceeds the tabulated value, the hypothesis that the decay times can be described as a Poisson process would be rejected at a particular confidence level. The calculations are given in Tables IV-1 through IV-3, and the results for χ^2 are given on each of the figures.

TABLE IV-1

GOODNESS OF FIT TESTS FOR GAMMA-DISTRIBUTED HALF-AMPLITUDE WAITING TIMES (Wn) IN RAYLEIGH WAVES RECORDED ON UNPROCESSED DIGITAL SEISMOGRAMS

W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	X^2
50	57	53.2	.272
100	41	37.2	.398
150	27	26.0	.039
200	20	18.2	.182
250	16	12.8	.800
300	11	8.9	.495
350	5	6.4	.307
400	4	4.3	.039
450	2	3.0	.333
≥500	1	2.1	.575
		X_9^2 =	3.599
Total Signals N		=	76
Mean Waiting Time m		=	140
Distance: less than 50 degrees			

W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	X^2
50	60	56.9	.169
100	52	47.6	.406
150	46	39.9	.934
200	38	33.3	.663
250	30	27.9	.158
300	27	23.2	.518
350	22	19.5	.321
400	17	16.3	.030
450	11	13.6	.496
500	10	11.4	.174
550	7	9.5	.659
≥600	6	7.9	.457
		X_{11}^2 =	4.986
		N =	68
		m =	280
greater than 50 degrees			

TABLE IV-2

GOODNESS OF FIT TESTS FOR GAMMA-DISTRIBUTED HALF-AMPLITUDE WAITING TIMES (Wn) IN RAYLEIGH WAVES RECORDED ON BANDPASSED DIGITAL SEISMOGRAMS

W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	X^2
50	92	87.0	.288
100	68	63.8	.278
150	50	46.6	.248
200	33	34.0	.029
250	24	25.0	.040
300	17	18.1	.067
350	13	13.3	.006
400	9	9.7	.051
450	6	7.1	.171
≥ 500	4	5.2	.277

X_9^2 = 1.455

Total Signals N = 119

Mean Waiting Time m = 160

Distance: less than 50 degrees

W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	X^2
50	87	85.2	.039
100	76	69.0	.710
150	65	57.4	.001
200	51	47.6	.242
250	41	39.6	.050
300	34	32.9	.037
350	28	27.4	.013
400	22	22.7	.022
450	19	18.9	.000
500	14	15.7	.184
550	10	13.0	.692
≥ 600	8	10.8	.725

X_{11}^2 = 3.715

N = 100

m = 270

greater than 50 degrees

TABLE IV-3

GOODNESS OF FIT TESTS FOR GAMMA-DISTRIBUTED HALF-AMPLITUDE WAITING TIMES (Wn) IN RAYLEIGH WAVES RECORDED ON PAPER RECORD SEISMOGRAMS

W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	X^2
50	58	56.6	.035
100	38	43.0	.582
150	28	32.5	.624
200	19	24.7	1.310
250	15	18.7	.733
300	13	14.1	.086
350	11	10.7	.009
400	9	8.1	.010
450	8	6.1	.593
500	7	4.7	1.125
550	5	3.5	.642
600	3	2.7	.034
650	2	2.1	.005
≥700	1	1.5	.167

$X^2_{13} = 5.955$

Total Signals N = 76

Mean Waiting Time m = 180

Distance: Less than 50 degrees

All of the comparisons of predicted and observed waiting time distributions show a "goodness of fit" value which leads to a "do not reject" for the hypothesis that the W_n are Poisson distributed at a .90 confidence level or greater. Confidence levels in the results range from about .925 in the paper record analysis to greater than .995 in the bandpassed signal analysis for events at less than 50 degrees. We therefore feel that the data are sufficiently random in a Poisson sense to proceed on with further analysis of the signal decay characteristics in this model.

The valuable characteristic of observing the half-amplitude times as we have and finding a reasonable fit to the Poisson model is that, by being able to describe the half-amplitude rates, the A/4, A/8,... rates should also be predicted by the gamma distribution method. If the time distribution of the first half-amplitude (A/2) is fit by the gamma parameter $r = 0$, then the waiting time from peak amplitude to second half amplitude (A/4) should be fit by $r = 1$, and so on, without change in the other parameters.

A demonstration of this characteristic is given for three cases where sufficient times between peak amplitude A and A/4 and A/8 were observed to attempt a comparison. The data are shown in Figures IV-4 and IV-5 for A/4 and A/8 amplitude increments, respectively. Figure IV-4 shows the observed distribution of the T_2 measurement for the signals recorded at less than 50 degrees epicentral distance. The calculated number of events expected was calculated from the gamma distribution using the parameter $r = 1$, and $\lambda = 1/m$, with m given in Figures IV-1, IV-2, and IV-3 for the respective half-amplitude rates. The X^2 statistic was also computed for the "goodness of fit" for each, and in each case the confidence in not rejecting the hypothesized model is greater than .950 (see Tables IV-4, IV-5).

Figure IV-5 shows the observed and predicted distribution of the number of events/waiting times for the T_3 measurement (time between peak amplitude and one-eighth amplitude) for the unprocessed and bandpassed signal

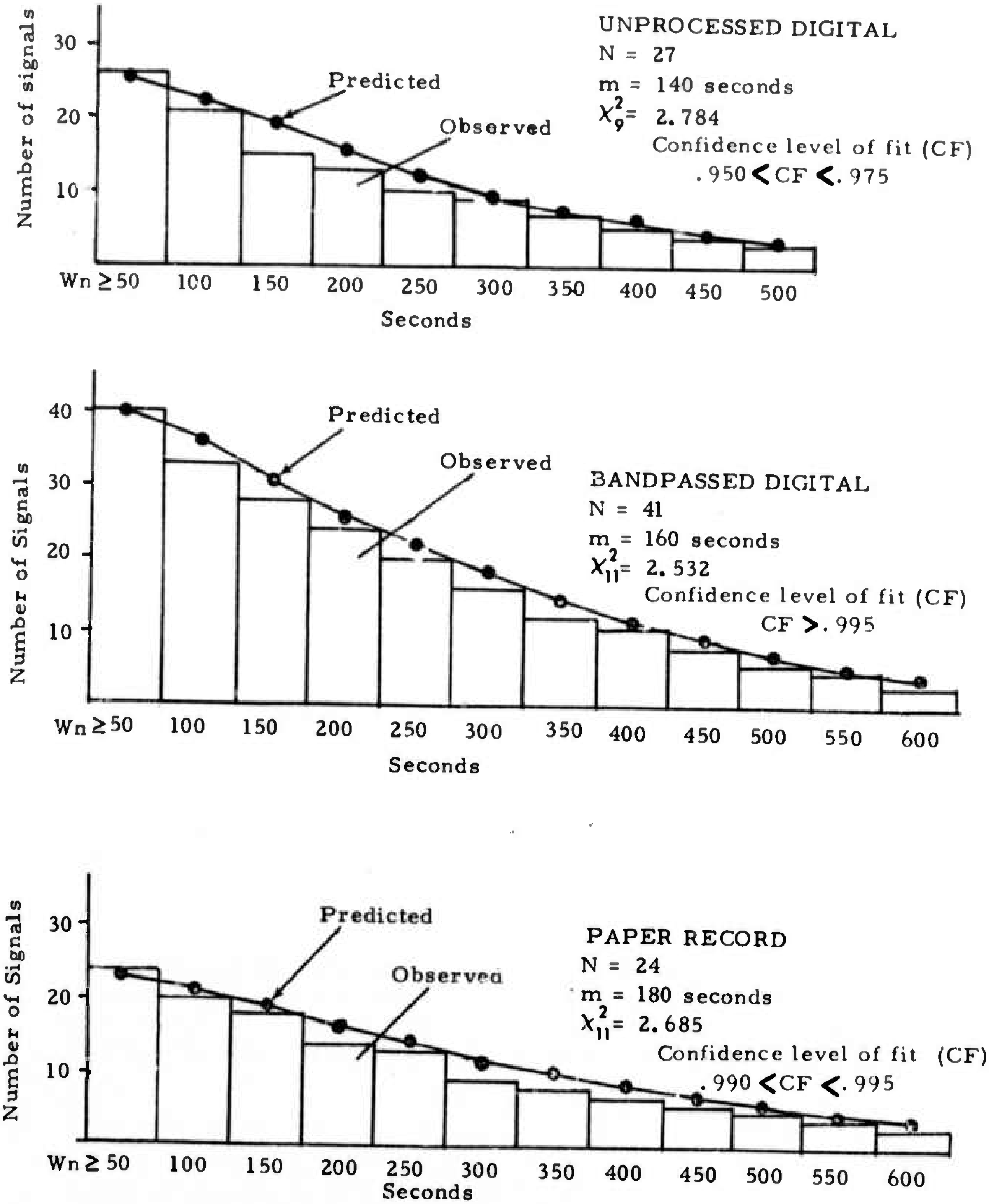


FIGURE IV-4

Predicted and Observed Gamma Distributed Waiting Times (Wn) For One-Quarter (A/4) Rayleigh Wave Amplitude Decay From Peak Signal Amplitude. Gamma Distribution Parameters Are r = 1 and Intensity 1/m. Epicentral Distance Less Than 50 Degrees.

UNPROCESSED DIGITAL

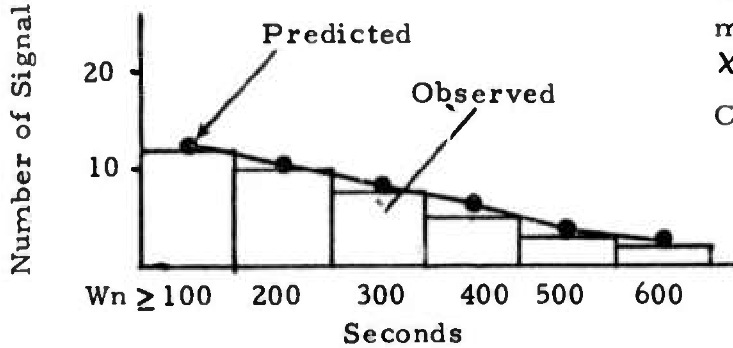
N = 13

m = 140 seconds

χ^2_5 = 0.842

Confidence level of fit (CF)

.950 < CF < .975



BANDPASSED DIGITAL

N = 22

m = 160 seconds

χ^2_7 = 1.630

Confidence level of fit (CF)

.975 < CF < .990

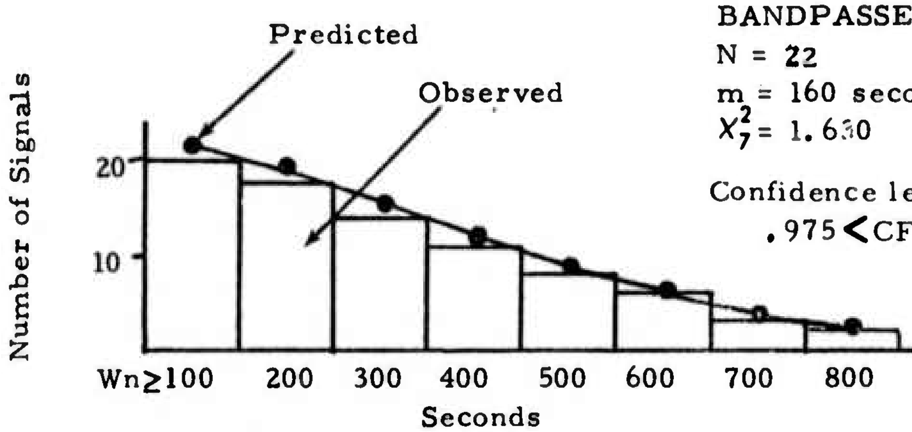


FIGURE IV-5

Predicted and Observed Gamma Distributed Waiting Times (Wn) For One-Eighth (A/8) Rayleigh Wave Amplitude Decay From Peak Signal Amplitude. Gamma Distribution Parameters Are r = 2 and Intensity 1/m. Epicentral Distance Less Than 50 Degrees.

TABLE IV-4

GOODNESS OF FIT TESTS FOR GAMMA-DISTRIBUTED ONE-QUARTER AMPLITUDE WAITING TIMES (W_n) IN RAYLEIGH WAVES RECORDED AT LESS THAN 50 DEGREES

Unprocessed Digital Seismograms

W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	χ^2
50	26	25.65	.005
100	21	22.68	.124
150	15	19.14	.895
200	13	15.69	.461
250	10	12.64	.551
300	9	9.91	.083
350	7	7.75	.073
400	5	6.78	.467
450	4	4.54	.064
≥500	3	3.46	.061

χ^2_9 = 2.784

Total Signals N = 27

Mean Waiting Time m = 160

Bandpassed Digital Seismograms

W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	χ^2
50	40	39.40	.009
100	33	35.71	.205
150	28	31.49	.389
200	24	26.40	.218
250	20	22.06	.192
300	16	18.16	.257
350	12	14.63	.472
400	11	11.76	.262
450	8	9.39	.206
500	6	2.46	.286
550	5	5.82	.115
≥600	3	4.59	.550

χ^2_{11} = .842

N = 41

m = 140

Paper Record Seismograms

W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	χ^2
50	24	23.23	.026
100	20	21.43	.095
150	18	19.10	.063
200	14	16.75	.451
250	13	14.33	.123
300	9	12.10	.794
350	8	10.10	.437
400	7	8.35	.218
450	6	6.89	.115
500	5	5.62	.068
550	4	4.85	.149
≥600	3	3.74	.146

χ^2 = 2.685

Total Signals N = 24

Mean Waiting Time m = 180

TABLE IV-5

GOODNESS OF FIT TESTS FOR GAMMA-DISTRIBUTED ONE-EIGHTH AMPLITUDE WAITING TIMES (Wn) IN RAYLEIGH WAVES RECORDED AT LESS THAN 50 DEGREES

Unprocessed Digital Seismograms				Bandpassed Digital Seismograms			
W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	X^2	W_n t (secs)	N obs. $W_n \geq t$	N pred. $W_n \geq t$	X^2
100	12	12.55	.024	100	20	21.47	.013
200	10	10.73	.050	200	18	19.07	.060
300	7	8.27	.195	300	14	15.69	.182
400	5	6.71	.435	400	11	11.95	.076
500	3	3.68	.038	500	8	8.73	.061
≥600	2	2.50	.100	600	6	6.17	.005
				700	3	4.25	.368
				≥800	2	2.70	.181
		X_5^2 =	.842			X_7^2 =	1.630
Total Signals N		=	13	Total Signals N		=	22
Mean Waiting Time m		=	140	Mean Waiting Time m		=	160

analysis. Insufficient data from the Paper Records were available for fitting this increment. The fit in this instance is not quite as "good" as above, but confidence that we should not reject the model fit is still very high (above .950). The fit is made using the gamma distribution with $r = 2$, and λ the same as in the $r = 0$ and $r = 1$ cases.

SECTION V
CONCLUSIONS

Demonstration of randomness of Rayleigh wave half-amplitude decay times, within constraints of wide spatial distribution of sources and recording stations (and at least a moderate range of source magnitudes), provides a basis for predicting signal amplitude - time relationships on seismograms. The two parameter gamma probability distribution describes the half-amplitude occurrence times adequately in a statistical sense. The parameter λ , which requires determination to use the distribution, can be directly derived from a set of observations of the half-amplitude times in recorded signals.

Practical use of the result given here lies in estimation of signal interference times expected in a network of recording stations during some typical period of operating time. Using the observed number of sources occuring (or expected) during such a period, distribution of the sources in terms of magnitude, and an acceptable amplitude-distance-magnitude relationship, the peak signal amplitudes can be estimated. Given these amplitudes, the time rate of amplitude decay by half-amplitude increments can be expressed at preselected probability levels. A continuation of the study will be concerned with this application.

A further continuation of the work includes an evaluation of the multiple source - single station signal amplitude decay times. If the randomness observed here is primarily a function of multiple paths and varying source magnitude, randomness of the Poisson type should also be found in the single station recordings with a sufficiently large base of observational data. The effects of the distance distribution of active seismic regions relative to the single stations can then be accounted for, and a more accurate estimate of potential interference times by Rayleigh waves in the network will result.

SECTION VI
REFERENCES

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